

## Improvement of Analytic Reconstruction Algorithms Using a Sinogram Interpolation Method for Sparse-angular Sampling with a Photon-counting Detector

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Sparse angular sampling has been studied recently owing to its potential to decrease the radiation exposure from computed tomography (CT). In this study, we investigated the analytic reconstruction algorithm in sparse angular sampling using the sinogram interpolation method for improving image quality and computation speed. A prototype of the spectral CT system, which has a 64-pixel Cadmium Zinc Telluride (CZT)-based photon-counting detector, was used. The source-to-detector distance and the source-to-center of rotation distance were 1,200 and 1,015 mm, respectively. Two energy bins (23~33 keV and 34~44 keV) were set to obtain two reconstruction images. We used a PMMA phantom with height and radius of 50.0 mm and 17.5 mm, respectively. The phantom contained iodine, gadolinium, calcification, and lipid. The Feld-kamp-Davis-Kress (FDK) with the sinogram interpolation method and Maximum Likelihood Expectation Maximization (MLEM) algorithm were used to reconstruct the images. We evaluated the signal-to-noise ratio (SNR) of the materials. The SNRs of iodine, calcification, and liquid lipid were increased by 167.03%, 157.93%, and 41.77%, respectively, with the 23~33 keV energy bin using the sinogram interpolation method. The SNRs of iodine, calcification, and liquid state lipid were also increased by 107.01%, 13.58%, and 27.39%, respectively, with the 34~44 keV energy bin using the sinogram interpolation method. Although the FDK algorithm with the sinogram interpolation did not produce better results than the MLEM algorithm, it did result in comparable image quality to that of the MLEM algorithm. We believe that the sinogram interpolation method can be applied in various reconstruction studies using the analytic reconstruction algorithm. Therefore, the sinogram interpolation method can improve the image quality in sparse-angular sampling and be applied to CT applications.

**Key Words:** Sinogram interpolation, Computed tomography (CT), Image reconstruction, Low-dose, Photon-counting detector

### Introduction

Computed tomography (CT) is widely used in the medical

field. Recently, sparse angular sampling in CT image reconstruction has been widely studied because it reduces patient radiation exposure and scanning time.<sup>1,2)</sup> Sparse-angular reconstruction does not offer high spatial resolution and it generates streak artifacts caused by inadequate projection data. However, it offers reasonable image quality, which extends the use of CT imaging to applications such as radiation therapy for treatment planning or industrial applications. Various reconstruction algorithms can be used to acquire reconstruction images in sparse tomographic imaging.<sup>4)</sup> In sparse-angle CT views, the iterative reconstruction algorithm has been extensively studied and developed.<sup>5)</sup> However, long reconstruction time makes it difficult to clinically apply the iterative re-

This research was financially supported by the Ministry of Trade, Industry & Energy (MOTIE), Korea Institute for Advancement of Technology (KIAT) and Gangwon Institute for Regional Program Evaluation (GWIRPE) through the Economic and Regional Cooperation Industry. Following are results of a study on the "Leades Industry-university Cooperation" Project, supported by the Ministry of Education (MOE). Received 7 July 2016, Revised 21 September 2016, Accepted 22 September 2016

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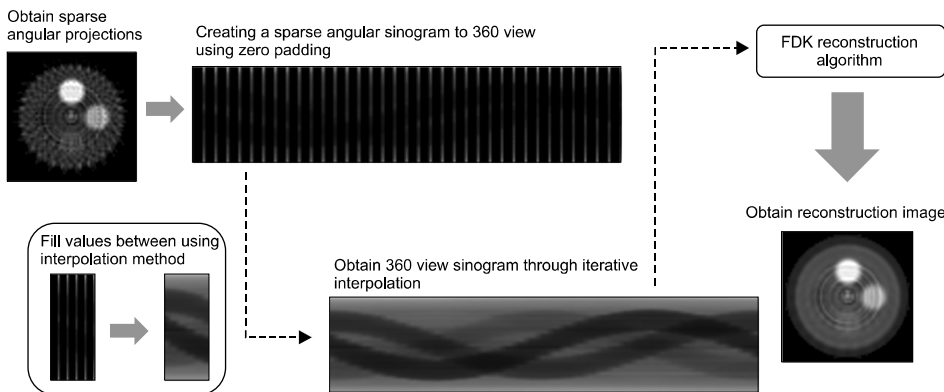
construction algorithm. Few studies have been conducted to obtain a reconstruction image in sparse angular sampling using the analytic reconstruction algorithm. In this study, therefore, we focused on the development of an analytical reconstruction algorithm in sparse tomographic imaging. We used the Feldkamp-Davis-Kress (FDK) algorithm with the sinogram interpolation method in an attempt to restore the lack of projection data to obtain a reasonable reconstruction image. There are various types of interpolation methods, including “cubic”, “spline”, “polynomial”, and “linear”.<sup>1,2)</sup> The proposed sinogram interpolation method was implemented with linear interpolation to calculate the incomplete sinogram information. As compared to the FDK algorithm alone, the sinogram interpolation method can minimize the noise and streak artifacts in reconstructed images.<sup>3)</sup> The sinogram interpolation method has been studied since CT was invented in the 1970s. Brooks et al. (1978) studied both interpolating new data in projection data and new projection data to attenuate Moire.<sup>2)</sup> In addition, Lahart (1981) studied a similar problem and used a least squares approximation to interpolate projection data when the external shape of the object was known (Marti Kalke et al. 2014).<sup>1,5)</sup>

The purpose of this paper was to evaluate the feasibility of the sinogram interpolation method using the FDK algorithm in sparse-angle view by means of the signal-to-noise (SNR).

## Materials and Methods

### 1. Sinogram interpolation method

The interpolation method was used to fill the insufficient projection data caused by sparse-angular sampling. A flowchart

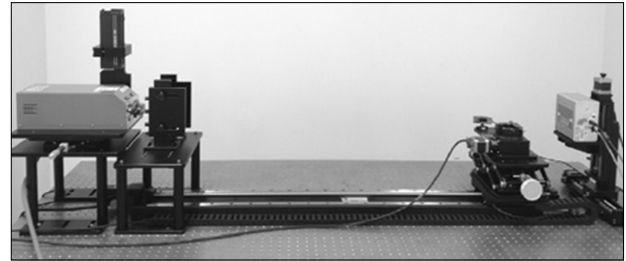


**Fig. 1.** Flowchart of the proposed sinogram interpolation method.

of the restoration process using the sinogram interpolation method is shown in Fig. 1. Initially, limited projection data were acquired in sparse-angular sampling. After obtaining these data, sparse angular sinogram images were acquired without applying the sinogram interpolation method using zero padding. In the second step, we filled in values using the interpolation method. We obtained restored sinograms by repeatedly filling in values. Finally, the restored projection data were reconstructed by using the FDK algorithm.

### 2. CZT-based photon-counting detector and spectral CT system

We used the photon-counting detector system prototype, which is widely used in spectral CT. Spectral CT has been studied in medical applications because of its energy separation capability.<sup>6,7)</sup> The spectral CT system consisted of a micro-focus X-ray tube (L8601-01, Hamamatsu, Japan). The source had a focal spot size of approximately  $5 \mu\text{m}$ . A high-precision motor-controlled rotary stage and Cadmium Zinc Telluride (CZT)-based photon-counting detector (eValuator-2500,



**Fig. 2.** Spectral CT system with CZT-based photon-counting detector.

eV Products, Saxonburg, PA, USA) were used in the spectral CT prototype system (Fig. 2). Table 1 lists the detailed acquisition parameters for obtaining the material-decomposed reconstruction images in the spectral CT system.

### 3. Phantom

A PMMA phantom with height and radius of 50 mm and 17.5 mm, respectively, was used. The phantom contained iodine, gadolinium, calcification, and liquid lipid, as shown in Fig. 3.

### 4. Reconstruction algorithm

In the experimental study, the Maximum Likelihood Expectation Maximization (MLEM) algorithm and the FDK algorithm were used to reconstruct the image. We compared two results of the MLEM algorithm and FDK algorithm with the sinogram interpolation method. The definitive aim of this

study was to verify the possibility of applying sinogram interpolation to sparse tomographic imaging. The FDK algorithm is one version of the filtered back projection algorithm that is widely used for CT reconstruction. The algorithm was originally proposed by Feldkamp Davis and Kress in 1984.<sup>9)</sup> Although this algorithm produces an approximate solution, it has been widely adopted for cone beam reconstructions.<sup>8,9)</sup> The equation for the FDK algorithm is

$$g(x,y,z) = \frac{1}{2} \int_0^{2\pi} d\beta \int_{-\infty}^{\infty} \frac{D^2}{(D-s)^2} R(p, \zeta, \beta) h\left(\frac{Dt}{D-s} - p\right) \frac{D}{\sqrt{D^2 + p^2 + \zeta^2}} dp$$

where  $g(x,y,z)$  is the value of the object at location  $(x,y,z)$ ,  $\beta$  is the angle relative to the  $z$ -axis,  $D$  is the source-to-isocenter distance,  $R(p, \zeta, \beta)$  is the projection data acquired at angle  $\beta$ ,  $t = x \cos \beta + y \sin \beta$ ,  $s = y \cos \beta - x \sin \beta$ , and  $(p, \zeta)$  represents the coordinate system of the detector.<sup>8)</sup> The MLEM reconstruction algorithm can be expressed as follows:

$$\lambda^{n+1} = \lambda^n \frac{\sum_j w_{ij} \frac{p_j}{\sum_i w_{ij} \lambda^n}}{\sum_i w_{ij}}$$

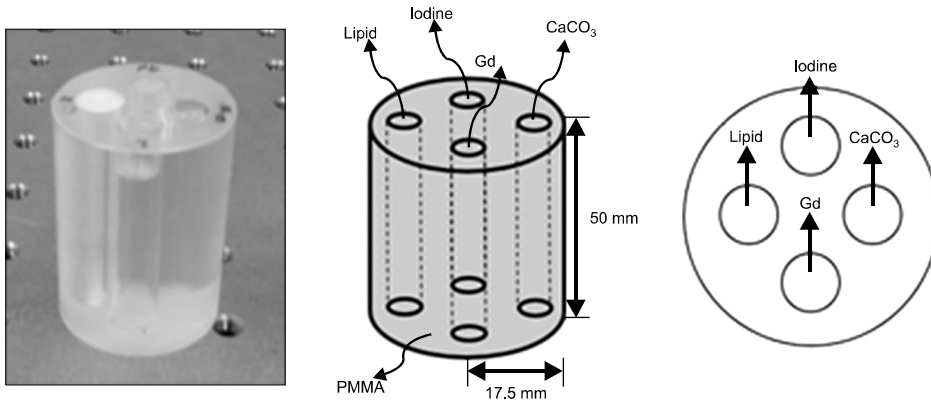
where  $\lambda^n$  is the current estimate of the image,  $p_j$  is the measured projection data, and  $w_{ij}$  is the probability that a photon emitted from image space at position  $i$  is detected by the detector pixel  $j$ .<sup>10,11)</sup> In this study, 10 iterations were executed for the convergence criterion of MLEM reconstruction algorithm.

### 5. Data analysis

The reconstructed image was a volume of  $128 \times 128 \times 128$  voxels. We set regions-of-interest (ROIs) on the 64th slice.

**Table 1. Detailed acquisition parameters of the spectral CT system.**

Source to detector distance (SID)	1,200 mm
Source to center of rotation distance	1,015 mm
Detector information	
Pixel pitch	0.8 mm
Detector length	51.2 mm
Number of pixels	64×1
Source information	
	bin 1: 23~33 keV
	bin 2: 34~44 keV
Number of projections (angle interval)	36 projections (10°)
Reconstruction method	
	1. Reconstruction method 'FBP'
	2. Filter 'Ram-lak'



**Fig. 3.** Illustration of the PMMA phantom containing four materials.

The reconstruction image was obtained using the lack of projection data in the sparse-angular sampling CT system. Four materials of square-shaped ROI regions were selected to evaluate the signal-to-noise ratio (SNR) in order to compare the methods with and without sinogram interpolation. The ROI regions were 15×15 pixels in size. The SNR was calculated as follows:

$$SNR = \frac{\mu}{\sigma}$$

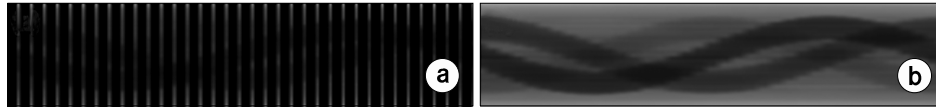
where  $\mu$  is the mean value of the average of material signal,

and  $\sigma$  is the standard deviation of the background.<sup>12)</sup>

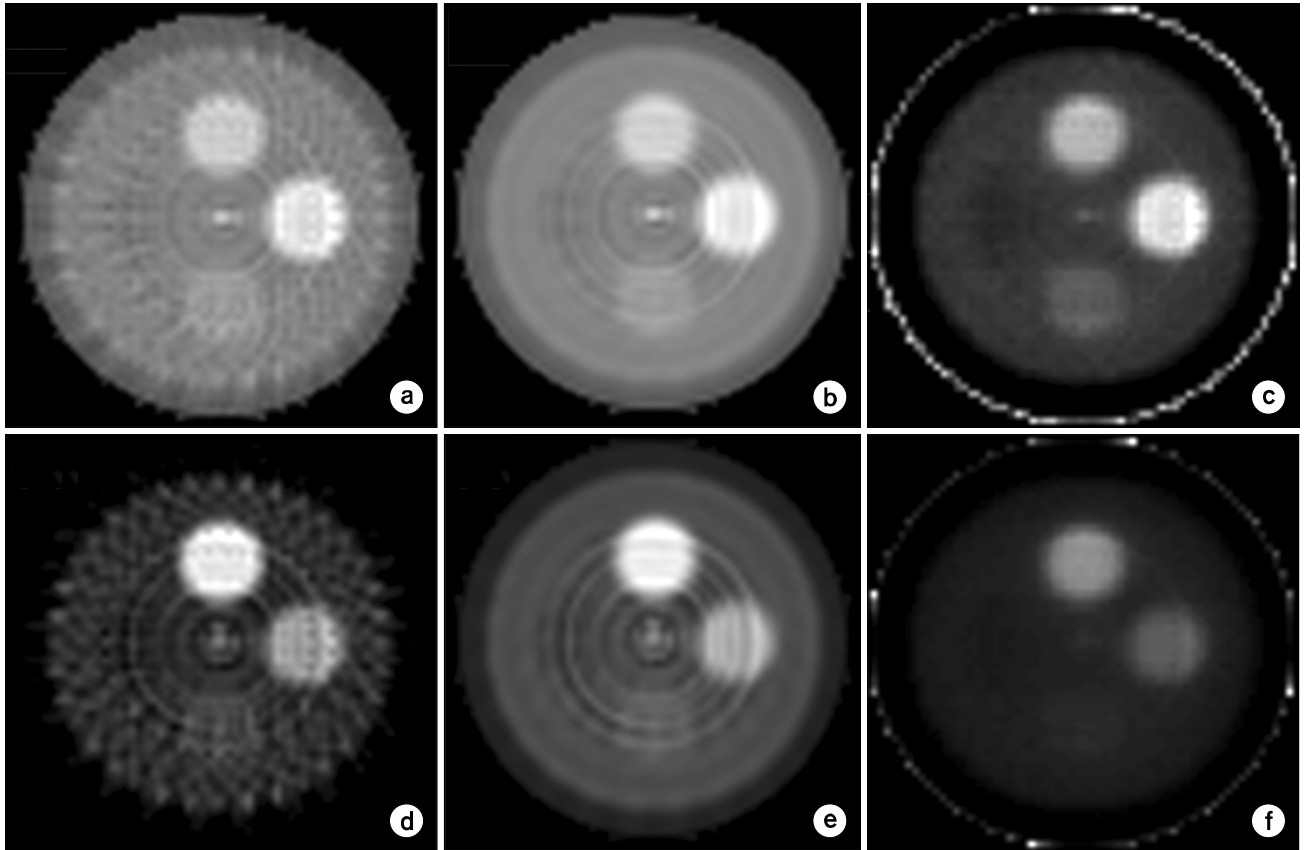
## Results

### 1. Results of images

We obtained the projection data of sparse-angular sampling. The sinogram interpolation method was implemented by linear interpolation to restore the incomplete sinogram information.<sup>1)</sup> Interpolation can be used to restore the sparse sampling sinogram data by filling values between, as shown in Fig. 4.



**Fig. 4.** Restored PMMA phantom sinogram. (a) Without interpolation in 36 views, (b) With interpolation in 36 angular views.



**Fig. 5.** The reconstruction images of the PMMA phantom with the FDK and MLEM algorithms. (a~c) Non-enhanced iodine image (23~33 keV): (a) FDK without the sinogram interpolation method, (b) FDK with the sinogram interpolation method, (c) MLEM (10 iterations). (d~e) Enhanced iodine image (34~44 keV): (d) FDK without the sinogram interpolation method, (e) FDK with the sinogram interpolation method, (f) MLEM (10 iterations).

Both the enhanced iodine (34~44 keV) and non-enhanced iodine (23~33 keV) reconstruction images were obtained using two energy bins with 36 projections, as shown in Fig. 5. The SNRs of iodine, calcification, and liquid lipid were increased by 167.03%, 157.93%, and 41.77%, respectively, with the 23~33 keV energy bin using the sinogram interpolation method. The SNRs of iodine, calcification, and liquid state lipid were also increased by 107.01%, 13.58%, and 27.39%, respectively, with the 34~44 keV energy bin using the sinogram interpolation method.

## 2. Results of the analysis

We conducted an experimental study in which we analyzed the SNR values of the reconstruction images with two energy bins. In the reconstructed images, the SNR values of FDK with the sinogram interpolation method were higher than those of FDK without the sinogram interpolation method, as presented in Figs. 6 and 7. The SNRs of iodine, calcification, and liquid lipid were increased by 167.03%, 157.93%, and 41.77%, respectively, with the 23~33 keV energy bin using the sinogram interpolation method. The SNRs of iodine, calcification, and liquid state lipid also increased by 107.01%, 13.58%, and 27.39%, respectively, with the 34~44 keV energy bin using the sinogram interpolation method. In most cases, the SNR value of MLEM was slightly higher than that of FDK with the

sinogram interpolation method.

The results of time consumption for reconstructing images by three methods are presented in Fig. 8. The MLEM reconstruction algorithm (10 iterations) took 60.7 s, which means that FDK with sinogram interpolation was 34 times faster than MLEM. Using FDK without sinogram interpolation was a less time consuming method than the other two methods. Although the reconstruction time of FDK with sinogram interpolation has been increased from 1.8 to approximately 21.6 s, it would provide better image quality.

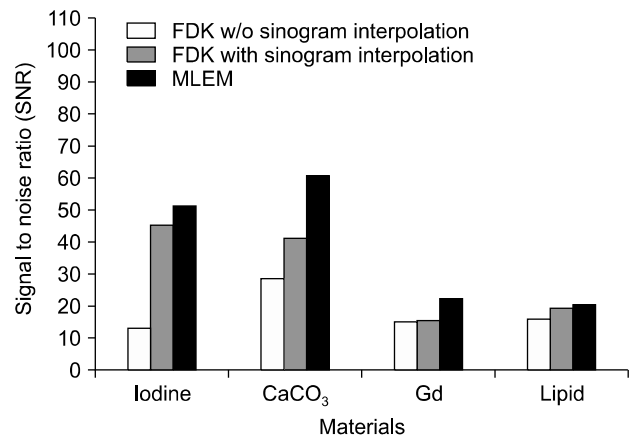


Fig. 7. The SNRs in reconstruction images by FDK with the sinogram interpolation method, FDK without the sinogram interpolation method, and MLEM (10 iterations) (34~44 keV).

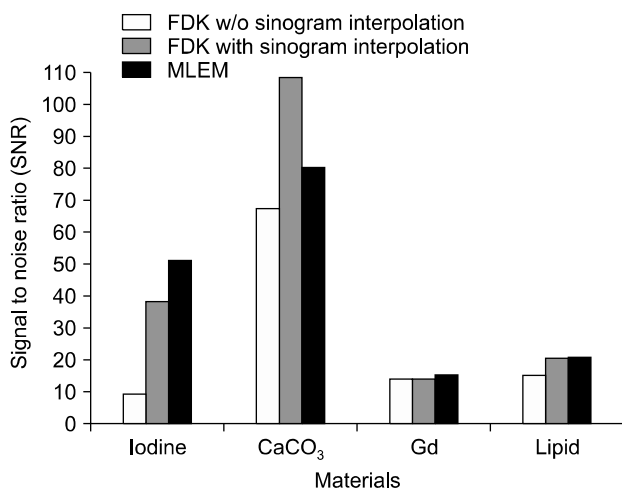


Fig. 6. The SNRs in the reconstruction images by FDK with the sinogram interpolation method, FDK without the sinogram interpolation method, and MLEM (10 iterations) (23~33 keV).

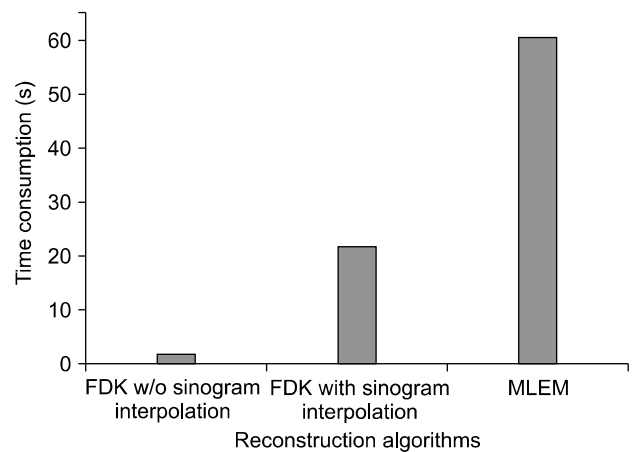


Fig. 8. Time consumption for reconstructing images by FDK with the sinogram interpolation method, FDK without the sinogram interpolation method, and MLEM (10 iterations).

## Discussion and Conclusion

In the real experimental study, the SNR values with the sinogram interpolation method were higher than those without the sinogram interpolation method. In two energy bins, the sinogram interpolation method increased the material signals and reduced the background noise. The FDK algorithm with the sinogram interpolation method did not result in better results than the MLEM algorithm in sparse-angular sampling. However, it produced comparable SNR results to those of the MLEM algorithm. These results suggest that the sinogram interpolation method can improve image quality of sparse-angular sampling.<sup>13,14)</sup>

Overall, the sinogram interpolation method can add value to the reconstruction process after sparse-angular sampling, which improves the image quality of the analytic reconstruction algorithm. Reconstructed images obtained using the sinogram interpolation method agreed well with those obtained using 360 projection data. By applying the sinogram interpolation to the FDK algorithm, we expect that the analytic reconstruction algorithm can obtain reasonable image quality of the iterative algorithm level. Furthermore, the analytic reconstruction algorithm using the sinogram interpolation method can be used in place of an iterative algorithm with a long reconstruction time. We believe that the sinogram interpolation method can be applied in various reconstruction studies using the analytic reconstruction algorithm.

In conclusion, our results suggest that the sinogram interpolation method can be useful in low dose CT applications such as radiation therapy for treatment planning or industrial CT applications.

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