

Caveats in Using Trotter and Gleser's (1958) Asian Equations for Stature Estimation

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Abstract : Trotter and Gleser's (1958) Asian equations have most frequently been used for stature estimation of Korean skeletal remains. However, limitations or caveats in using those equations have rarely been argued. This study reviews five issues frequently overlooked in applying the equations and interpreting the outcomes.

First, Trotter and Gleser (1958) multiply the standard errors by 2 to obtain the 95% prediction interval (PI). However, there is discrepancy between their calculation and actual PIs, and thus correct method for PI calculation is recommended. Secondly, given the uncertainty about the tibia length measurement, there is a possibility that the tibia-related equations yield biased estimates. Thirdly, since a mathematical error was incorporated in the development process of the combined equations, caution should be taken. Fourthly, extrapolation may cause additional unexpected error in applying regression equations. Therefore, it is necessary to check if a target sample falls within the reference sample range prior to using the equations. Lastly, applying Trotter and Gleser's (1958) equations to female samples should be avoided because they produce highly biased estimates.

The issues discussed in this study will contribute to reducing potential errors associated with application of these equations, and eventually enhancing accuracy of the final stature estimates. If errors are anticipated but unavoidable, discussions about the potential errors should be made to minimize misunderstanding about the outcomes.

Keywords: Trotter and Gleser (1958), Asian equation, Stature estimation, Korean, Skeletal remains

Introduction

Stature has extensively been studied as an indicator of health, environmental condition, as well as socio-economic and political circumstances of an individual or population in various fields including paleoanthropology, bioarchaeology, and physical anthropology [1-8]. In addition, stature consists of a crucial aspect of biological profile of uniden-

tified victims in a forensic context [9-13].

Stature estimation techniques are generally grouped into two categories: anatomical and mathematical methods [14,15]. The anatomical methods reconstruct stature by summing up the lengths/heights of all bone elements contributing to a standing height, and taking soft tissue corrections into account [12,15-17]. On the other hand, the mathematical methods produce stature estimates using regression equations based on a high correlation between stature and specific body parts [9,13,18]. The accuracy of the anatomical method is reportedly higher than the mathematical method [11,14,15] because, unlike the mathemati-

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cal method, it does not require any assumptions regarding body proportions [14,15]. Nevertheless, the applicability of the anatomical methods is often limited because they require complete or nearly complete skeletons [9,14,15]. Therefore, the mathematical methods utilizing only a small number of elements are greatly useful when the remains are in a poor condition [9]. However, it should also be noted that the accuracy of the estimates depends on three issues: whether the equations is based on appropriate reference samples, what bone elements are used, and on what type of regression model are the equations based [14,15].

Trotter and Gleser's [13,19] equations are one of the popular mathematical methods. In 1952, Trotter and Gleser [13] devised new equations for American Whites and Blacks¹. They used the U.S. World War II casualties and skeletal samples from the Terry Anatomical Skeletal collection for male and female equations, respectively. Six years later, using the U.S. Korean War casualties, Trotter and Gleser [19] presented another set of male equations for Asians² and Mexicans in addition to American Whites and Blacks. Unlike the 1952 study where equations were generated using average lengths of paired bones, the 1958 study used left and right bones separately.

Trotter and Gleser's 1958 equations have been widely utilized particularly in non-Western countries [20,21]. Based on the preliminary literature review, these equations are the most commonly used technique for stature estimation in Korea. Moreover, the consistency in body proportions of Korean populations through time [20] makes the accuracy of these equations independent of the time periods, to which skeletal remains belong, and has allowed the equations to be applied to both archaeological and forensic settings. Nevertheless, limitations or caveats regarding application of the equations have rarely been argued. The purpose of this study is to review five issues frequently overlooked in using the equations and interpreting the outcomes. This discussion will eventually contribute to enhance accuracy of the final stature estimates by reducing potential errors associated with application of these equations.

Methods

Careful reading of the Trotter and Gleser's [19] original paper and related research articles raised three issues regarding the generation process of the equations: (1) misinterpretation of standard errors, (2) mis-measurement of tibia length, (3) Mathematical error in producing the combined equations. Two additional issues with regard to practices of the equations were noticed while reviewing 33 references published in Korea between the 1960s and 2010s: (1) application to extrapolation/extreme cases, and (2) application to female samples. These references were randomly selected by the first author (Appendix 2). Each of these issues is discussed in the following chapter.

Results and Discussion

1. Misinterpretation of standard errors

In interpreting standard errors (SE) associated with their equations, Trotter and Gleser [19] explain that "it could be stated with 95% certainty that the actual stature had been between" [a point estimate $- 2 \times \text{SE}$] and [a point estimate $+ 2 \times \text{SE}$] (p. 117). However, as Wilson et al. [10] claim, the SE is not a predictive estimator but a point estimator indicating an overall discrepancy between actual values and estimated values around a regression line within a reference sample. It does not provide information about a specific X value but general information about the whole reference sample associated with the equation. Thus, it is inappropriate to directly relate a specific bone length to a specific stature range using the SE. Rather, the prediction interval (PI) should be utilized for this purpose because the PI is calculated based on specific X values [22]. In the case of regression equation with a small sample size and large SE, the PI tends to become wide. Also, X values around the mean of X s (\bar{X}) have narrow PIs compared to those away from the mean. Therefore, PIs are expressed as a hyperbola with a narrow width around the \bar{X} in the Cartesian coordinates [22].

Literature review of Korean references showed that

^{1,2}In the original paper, Trotter and Gleser (1958) used the terms 'Negro' and 'Mongoloid' to indicate the African and Asian ancestries. However, in this study, the terms 'Negro' and 'Mongoloid' were replaced by 'Black' and 'Asian', respectively.

researchers tend to report point estimates and associated SEs separately without further interpretations. When range estimates were reported, most of them were calculated without doubling the SEs, which is not concordant with the Trotter and Gleser's [19] instruction. PIs were not utilized in any literature reviewed in this study. This practice may cause misunderstanding about true prediction error range or prediction interval. In fact, it should be admitted that even when researchers strictly follow the Trotter and Gleser's [19] suggestion (i.e., doubling SEs to yield range estimates), unexpected errors are likely to be produced due to the nature of the SEs themselves. Therefore, to avoid unnecessary confusion, an effort to calculate and report the PIs is recommended [22].

2. Mis-measurement of tibia length

Trotter and Gleser [19] explain that the tibia length should be measured following the description presented in their 1952 study, where the medial malleolus is included (see A in Fig. 1). However, as Jantz et al. [23,24] clearly demonstrated, the authors did not include the malleolus length in tibia measurements in their 1952 study, which resulted in underestimation of the tibia length by 13.6 mm on average. Moreover, bone measurements in their 1958 study were not taken by the authors but other technicians [19]. Although a significant increase in the tibia length was noticed in the 1958 study compared to the 1952 study, the authors attributed the difference to inter-observer errors and an intrinsic difficulty of reproducing tibia measurements rather than inclusion of a malleolus length [19]. Therefore, it is still uncertain whether the malleolus was included in the 1958 study.

Due to the ambiguity of the measurement method, caution is required when using Trotter and Gleser's [13,19] tibia equations. As Jantz et al. [24] recommend, avoiding the tibia equations will be a preferable option if other long bones are available. However, in the case of the tibia equations being used, researchers should clarify detailed measurement method and make comments about possible errors related to the tibia measurement in Trotter and Gleser [13,19].

3. Mathematical error in producing the combined equations

Between the side-specific and side-combined equations

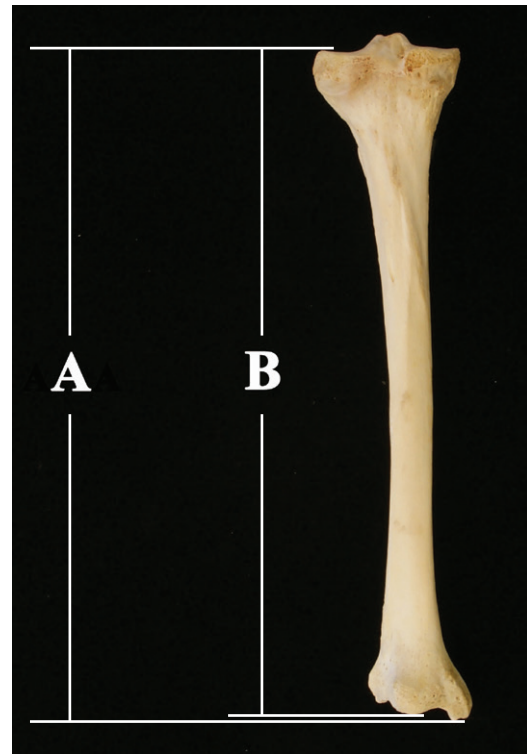


Fig. 1. (A) tibia maximum length by Trotter and Gleser's [13,19] definition; (B) actual length measured by Trotter and Gleser [13].

presented in Trotter and Gleser [19], the latter has been more popularly used partly because Trotter [25] listed only the combined ones as recommendable equations.

Trotter and Gleser [19] generated the combined equations using side-specific equations developed in the same study instead of using actual average lengths of bones. The slopes of combined equations were calculated by averaging the slopes of the left and right bone equations. Then, they averaged the means of right and left bone lengths to represent a point in the Cartesian coordinates, through which a line with the slope calculated as above should pass [19].

However, in theory, averaging slopes of the left and right bone equations does not yield the slope that would be obtained given actual average bone lengths. In Fig. 2, except at the intersecting point, every single value on the Y axis corresponds to two X values. Line 1 connects the midpoints of the two Xs, which represents the relationship between statures and the average lengths of the left and right bones. However, Trotter and Gleser's [19] combined equations yield Line 2, which bisects two Ys corresponding to a common X value, by averaging two slopes of the

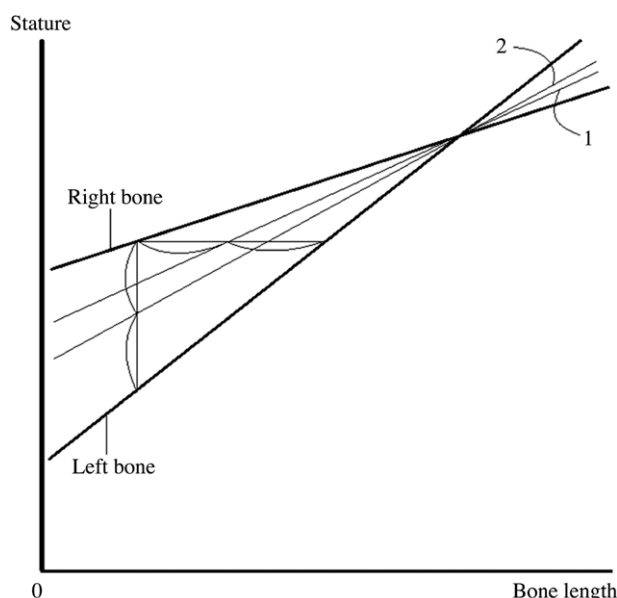


Fig. 2. Exemplified diagram showing the relationship between the regression line where statures are regressed on actual average bone lengths (Line 1) and the regression line representing Trotter and Gleser's [19] combined equation (Line 2).

left and right bone equations. Since Line 2 always have a larger slope than Line 1 (see Appendix 1 for mathematical proof), the combined equations underestimate statures for the bones shorter than the intersecting point of the left and right equations, and vice versa. Using side-specific equations instead of the combined equations will be a preferable alternative to avoid this error. Otherwise, the amount of error should be calculated based on the bone lengths and taken into account in the final stature estimates.

4. Application to extrapolation/extreme cases

Since Pearson's work in 1899, linear regression method has been the most popular way to develop stature estimation equations [13,19,21,26-31]. In general, under an assumption of linearity between variables, linear regression analysis 1) describes a pattern of the relationship in a numeric form and/or 2) predicts one variable from another variable. Trotter and Gleser's [13,19] equations were also generated using the linear regression analysis for a prediction purpose.

In using linear regression equations, extrapolation should be avoided [32,33]. Extrapolation refers to a situation where a Y value is predicted using an X value that is off the range of a reference sample. Extrapolation may

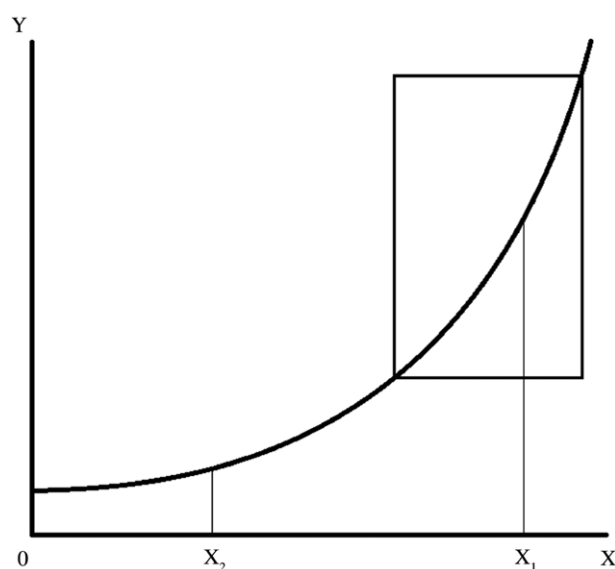


Fig. 3. Exemplified diagram showing two variables of a curvilinear relationship. Note that, despite their true relationship, the graph in a boxed range is nearly linear.

result in a biased result because the relationship between X and Y variables is not necessarily linear outside of the reference sample.

Suppose two variables have a curvilinear relationship as seen in Fig. 3. If the box denotes a reference sample range, the relationship between variables will be nearly linear in the boxed area regardless of the true relationship. Thus, a Y value corresponding to X_1 can be estimated using a linear regression equation derived from the boxed area. However, a significant error will occur if a Y value corresponding to X_2 is estimated using the same equation, because the two variables do not have a linear relationship outside of the reference sample range any more.

Even in the case of interpolation, accuracy of an estimate may be compromised as corresponding X s depart from the mean of a reference sample. Extreme cases yield highly biased estimates particularly when the ordinary least squares (OLS) model is utilized compared to other regression methods [34]. Therefore, it is necessary to check if a target sample is an extreme or extrapolation case with regard to a reference sample by carefully reviewing descriptive statistics of the reference samples.

Table 1 presents part of the information for the Asian samples in Trotter and Gleser [19]. The 95% ranges in Table 1 were calculated using the descriptive statistics provided in the original paper. These ranges may be used

Table 1. Bone lengths of the Asian samples in Trotter and Gleser [19].

Bone (side)	Asian sample used in Trotter and Gleser [19]		
	<i>n</i>	Mean \pm S.D. (cm)	95% range (cm)
Humerus (right)	74	31.768 \pm 1.857	28.128~35.408
Humerus (left)	65	31.742 \pm 1.836	28.143~35.341
Radius (right)	68	24.547 \pm 1.497	21.613~27.481
Radius (left)	67	24.303 \pm 1.460	21.441~27.165
Ulna (right)	65	26.251 \pm 1.535	23.242~29.260
Ulna (left)	65	26.126 \pm 1.560	23.068~29.184
Femur (right)	67	44.246 \pm 2.479	39.387~49.105
Femur (left)	60	44.640 \pm 2.476	39.787~49.493
Fibula (right)	61	36.146 \pm 2.170	31.893~40.399
Fibula (left)	62	36.340 \pm 2.273	31.885~40.795

Table 2. Magnitude of errors by applying Trotter and Gleser's [19] equations to Korean female samples [36].

Error	Bone dimension					
	FeL ³	TiL ⁴	FeL + TiL	HuL ⁵	RaL ⁶	HuL + RaL
D ¹ (cm)	-6.5	-6.7	-6.3	-7.8	-5.2	-5.6
% PE ²	-4.2	-4.3	-4.1	-5.0	-3.7	-3.4

¹Mean difference = mean of [true stature - estimated stature]

²Percent prediction error = mean of [(True stature - Estimated stature) \times 100/Estimated stature]

³Maximum length of the femur

⁴Condylar-malleolus length of the tibia

⁵Maximum length of the humerus

⁶Maximum length of the radius

to determine if a target sample falls in an extrapolation/extreme case in terms of the reference sample. In case of extrapolation/extreme samples being estimated by these equations, potential errors associated with the estimates should be clearly discussed. Otherwise, it is recommended to find alternative methods rather than using Trotter and Gleser [19].

5. Application to female samples

Trotter and Gleser did not provide female equations in their 1958 study. Thus, in the U.S., stature of Asian females is often estimated using the female equations of other ancestry (e.g., equations for White [13] or Mesoamerican females [30]) rather than using Trotter and Gleser's [19] male equations (Jantz, personal communication). However, literature review in this study exhibited that Trotter and Gleser [19] has frequently been utilized for both males and females in Korea.

In general, male equations yield biased estimates for female samples due to the allometry between stature and long bones, and a sexual dimorphism in body proportion

[35]. Jeong et al. [36] demonstrate that Trotter and Gleser's [19] male equations overestimate Korean female statures as much as 7.8 cm (Table 2), which is bigger than the bias female equations from other studies are likely to produce [37]. The authors explain that the overestimation was possibly due to a combinatory effect of the extrapolated application of the equations and the allometric nature of the long bones. The average stature of the Korean female samples used in Jeong et al. [36] (148.7 cm) was significantly lower than Trotter and Gleser's [19] reference sample (168.73 cm), and most samples fell in the left side of extreme or extrapolation cases. Moreover, the slopes of the Trotter and Gleser's [19] equations were smaller than the actual relationship between stature and Korean female bone lengths due to an allometric effect [36]. Therefore, small individuals tend to exhibit more biased estimates.

Considering the magnitude of bias, applying Trotter and Gleser's [19] equations to Korean female samples is not recommended. Instead, Korean specific equations for females should be utilized to minimize associated errors [37-39].

Conclusion

Researchers do not ‘determine’ but ‘estimate’ statures. In the process of estimation, errors due to natural variations of human properties are inevitable. However, the errors associated with generation and application of equations should be minimal. In order to minimize unexpected errors and avoid confusion regarding stature estimates using Trotter and Gleser’s [19] Asian equations, five issues were discussed in this study. Some issues (misinterpretation of standard errors, mis-measurement of tibia length, and mathematical error in producing the combined equations) are intrinsically associated with the generation process of the equations, which means that errors are unlikely to be removed even when the authors’ instruction is carefully followed. In this case, researchers need to clearly point out these issues and discuss the limitations in their reports to avoid further misunderstanding or confusion. Unexpected errors also occur in the process of application of the equations by expanding the applicability of the equations arbitrarily (extrapolation/extreme cases and application to female samples). This practice compromises accuracy of the final estimates. Moreover, when a compounding error occurs, interpretation of the results becomes more problematic. In this case, it is recommended to calculate and report potential errors along with final estimates with caution. Otherwise, choosing an alternative method may be more appropriate.

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Trotter와 Gleser (1958)의 아시아인 공식을 사용하여 한국인 키를 추정할 때 유의할 점에 대한 고찰

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간추림 : 한국인 뼈가 출토되었을 때 그 사람의 생전 키를 추정하기 위해 Trotter와 Gleser (1958)의 아시아인 공식이 빈번히 사용된다. 하지만 이 공식을 적용하고 그 결과를 해석함에 있어서 어떤 점에 유의해야 하는지, 혹은 이 공식 자체가 가진 한계에 대한 논의는 많이 이루어지지 않고 있다. 본 연구에서는 이 공식을 사용하는 연구자들이 쉽게 간과함으로써 결과의 정확성을 떨어뜨리거나 결과에 대한 오해를 불러올 수 있는 5가지 사안에 대해 주의를 환기하고자 하였다.

Trotter와 Gleser는 표준오차(standard error)에 2를 곱함으로써 95% 신뢰도의 예측범위(prediction interval)를 구하고자 하였으나 실제 95% 예측범위는 이와 동일하지 않으므로 주의를 요한다. 또한 그들의 연구에 사용된 정강뼈의 계측 방법에 대한 논란이 있는 만큼 정강뼈가 포함된 공식을 사용할 때는 오류의 가능성을 염두에 두어야 한다. 일반적으로 사용되는 Trotter와 Gleser (1958)의 좌우측 평균치 공식(combined equations)은 실제 좌우측 뼈의 평균치를 이용하는 대신 좌측뼈 공식과 우측뼈 공식을 사용하여 개발되었으며 이로 인해 뼈 길이에 따른 일정 정도의 오류가 불가피하게 된다. Trotter와 Gleser (1958)의 공식은 선형회귀분석 방법을 기반으로 개발되었기 때문에 이 연구에 사용된 자료와 비교해 지나치게 크거나 작은 뼈를 이용하여 키를 추정하게 되면 예측하지 못한 오류가 추가로 발생하게 된다. 마지막으로 Trotter와 Gleser (1958)는 남성 공식만을 개발했는데 만약 이 남성 공식을 여성의 뼈에 적용하게 되면 실제 키보다 지나치게 크게 추정하게 되므로 이 공식을 여성 뼈에 적용해서는 안된다.

이 5가지 사안을 염두에 둔다면 Trotter와 Gleser (1958)의 공식을 사용하여 결과를 도출하고 해석함에 있어서 오류를 최소화할 수 있을 것이다. 특히 Trotter와 Gleser의 설명을 정확히 따르는 경우에도 불가피하게 오류가 발생할 수 있는 만큼 연구자들은 최종 결과와 함께 추가적인 오류의 가능성을 논함으로써 결과에 대한 불필요한 오해를 줄여나가야 할 것이다.

찾아보기 낱말 : Trotter and Gleser (1958), 키 추정 공식, 오류, 한국인, 사람뼈

Appendix 1. Mathematical error associated with the process of combined equation development in Trotter and Gleser [19].

There is a theoretical error in the process of combined equation development in Trotter and Gleser [19]. For a proof purpose, two slopes are compared each other: one is the slope presented in Trotter and Gleser [19] and the other is the true slope which would be obtained given actual averages of paired bones. In Fig. A-1, suppose that Y_1 and Y_2 denote statures of individuals 1 and 2, respectively. In addition, X_{1R} (X_{2R}) and X_{1L} (X_{2L}) denote right and left bone lengths of the individual 1 (2), respectively. For the convenience sake, it is assumed that all bone lengths lie on the regression lines and individual 2 is bigger than individual 1 (i.e., $Y_2 > Y_1$, $X_{2L} > X_{1L}$, and $X_{2R} > X_{1R}$).

In the Cartesian coordinates, slope of a linear line is defined as the ratio of Y 's variation to X 's variation (i.e., $\Delta Y/\Delta X$). Therefore, the slopes of the right and left bone equations can be calculated as below;

$$S_{\text{Right bone equation}} = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_{2R} - X_{1R}}$$

$$S_{\text{Left bone equation}} = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_{2L} - X_{1L}}$$

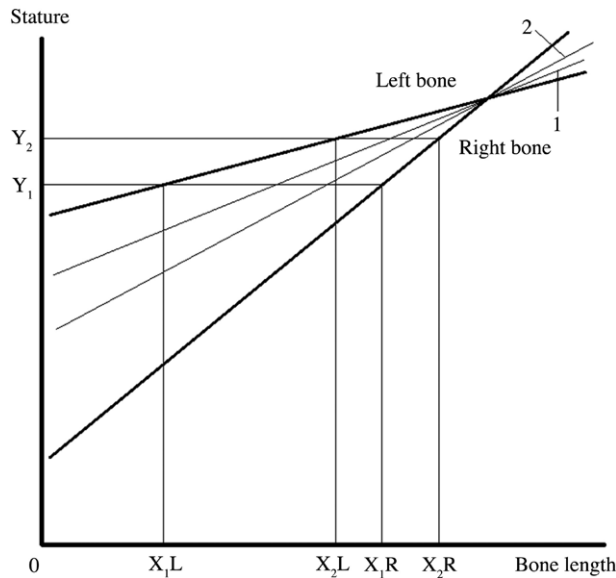


Fig. A-1. Exemplified diagram showing the relationship between two individuals. Individual 1 with a stature Y_1 possesses left and right bones of X_{1L} and X_{1R} ; Individual 2 with a stature Y_2 possesses left and right bones of X_{2L} and X_{2R} .

Trotter and Gleser [19] averaged two slopes from side-specific equations to yield a slope of combined equation, which can be expressed as below;

$$\begin{aligned} S_{\text{Combined equation}} &= \text{Arithmetic mean of } S_{\text{Right bone equation}} \text{ and } S_{\text{Left bone equation}} \\ &= \frac{1}{2} \left(\frac{Y_2 - Y_1}{X_{2R} - X_{1R}} + \frac{Y_2 - Y_1}{X_{2L} - X_{1L}} \right) \\ &= \frac{(Y_2 - Y_1) \{ (X_{2L} - X_{1L}) + (X_{2R} - X_{1R}) \}}{2(X_{2R} - X_{1R})(X_{2L} - X_{1L})} \end{aligned} \quad (1)$$

On the other hand, the true slope should be obtained using the average bone length of individual 1 (i.e., $\frac{1}{2}(X_{1L} + X_{1R})$) and that of individual 2 (i.e., $\frac{1}{2}(X_{2L} + X_{2R})$). Since the regression line should pass the two points, $[\frac{1}{2}(X_{1L} + X_{1R}), Y_1]$ and $[\frac{1}{2}(X_{2L} + X_{2R}), Y_2]$, the true slope is calculated as below;

$$\begin{aligned} S_{\text{true}} &= \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{\frac{1}{2}(X_{2L} + X_{2R}) - \frac{1}{2}(X_{1L} + X_{1R})} \\ &= \frac{2(Y_2 - Y_1)}{(X_{2L} - X_{1L}) + (X_{2R} - X_{1R})} \end{aligned} \quad (2)$$

When substituting A for $(X_{2L} - X_{1L})$, B for $(X_{2R} - X_{1R})$, and C for $(Y_2 - Y_1)$, A, B, and C should have positive values because of the assumption that $X_{2L} > X_{1L}$, $X_{2R} > X_{1R}$, and $Y_2 > Y_1$. Then, which slope is larger can be determined by subtracting formula 2 (i.e., true slope) from formula 1 (i.e., combined-equation slope) as below.

$$\begin{aligned} (1) - (2) &= \frac{C(A+B)}{2AB} - \frac{2C}{A+B} \\ &= \frac{C\{(A+B)^2 - 4AB\}}{2AB(A+B)} = \frac{C(A-B)^2}{2AB(A+B)} \end{aligned} \quad (3)$$

In the formula 3, the denominator always has a positive value because A, B, and C are all positive. For the same reason, the numerator is always positive except for the case of $A = B$. In other words, unless A is equal to B, formula 1 (i.e., combined-equation slope) is always larger than formula 2 (i.e., true slope). Yet, in fact, the situation where A is equal to B is quite unlikely because it presupposes that a difference between left bones (i.e., $X_{2L} - X_{1L}$) is always the same as a difference between right bones (i.e., $X_{2R} - X_{1R}$) in any two individuals. Therefore, it can be concluded that the slope of the Trotter and Gleser's [19] combined equations is always bigger than the true slope where actual average lengths of bones are used.

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